

# **GCSE Mathematics (1MA1)**

## **Higher Tier**

### **Scheme of Work**

Unit		Title	Estimated hours
1	<u>a</u>	Calculations, checking and rounding	4
	<u>b</u>	Indices, roots, reciprocals and hierarchy of operations	4
	<u>c</u>	Factors, multiples, primes, standard form and surds	7
2	<u>a</u>	Algebra: the basics, setting up, rearranging and solving equations	10
	<u>b</u>	Sequences	4
3	<u>a</u>	Averages and range	4
	<u>b</u>	Representing and interpreting data and scatter graphs	5
4	<u>a</u>	Fractions and percentages	12
	<u>b</u>	Ratio and proportion	6
5	<u>a</u>	Polygons, angles and parallel lines	6
	<u>b</u>	Pythagoras' Theorem and trigonometry	6
6	<u>a</u>	Graphs: the basics and real-life graphs	6
	<u>b</u>	Linear graphs and coordinate geometry	8
	<u>c</u>	Quadratic, cubic and other graphs	6
7	<u>a</u>	Perimeter, area and circles	5
	<u>b</u>	3D forms and volume, cylinders, cones and spheres	7
	<u>c</u>	Accuracy and bounds	5
8	<u>a</u>	Transformations	6
	<u>b</u>	Constructions, loci and bearings	7
9	<u>a</u>	Solving quadratic and simultaneous equations	7
	<u>b</u>	Inequalities	6
10		Probability	8
11		Multiplicative reasoning	8
12		Similarity and congruence in 2D and 3D	6
13	<u>a</u>	Graphs of trigonometric functions	6
	<u>b</u>	Further trigonometry	9
14	<u>a</u>	Collecting data	4
	<u>b</u>	Cumulative frequency, box plots and histograms	6
15		Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics	7
16	<u>a</u>	Circle theorems	5
	<u>b</u>	Circle geometry	5
17		Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof	7
18		Vectors and geometric proof	9
19	<u>a</u>	Reciprocal and exponential graphs; Gradient and area under graphs	7
	<u>b</u>	Direct and inverse proportion	7

**UNIT 1: Powers, decimals, HCF and LCM, positive and negative, roots, rounding, reciprocals, standard form, indices and surds**
**1a. Calculations, checking and rounding**  
 (N2, N3, N5, N14, N15)

**Teaching time**  
 3-5 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Add, subtract, multiply and divide decimals, whole numbers including any number between 0 and 1;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use the product rule for counting (i.e. if there are  $m$  ways of doing one task and for each of these, there are  $n$  ways of doing another task, then the total number of ways the two tasks can be done is  $m \times n$  ways);
- Round numbers to the nearest 10, 100, 1000, the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

**1b. Indices, roots, reciprocals and hierarchy of operations**  
 (N2, N3, N6, N7, N14)

**Teaching time**  
 3-5 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Use index notation for integer powers of 10, including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Recall that  $n^0 = 1$  and  $n^{-1} = \frac{1}{n}$  for positive integers  $n$  as well as,  $n^{\frac{1}{2}} = \sqrt{n}$  and  $n^{\frac{1}{3}} = \sqrt[3]{n}$  for any positive number  $n$ ;
- Understand that the inverse operation of raising a positive number to a power  $n$  is raising the result of this operation to the power  $\frac{1}{n}$ ;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
- Solve problems using index laws;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
- Use an extended range of calculator functions, including  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $x^2$ ,  $\sqrt{x}$ , memory,  $x^y$ ,  $x^{\frac{1}{y}}$ , brackets;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

**1c. Factors, multiples, primes, standard form and surds**  
(N3, N4, N8, N9)**Teaching time**  
6-8 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF and LCM, and prime numbers;
- Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors;
- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form;
- Understand surd notation, e.g. calculator gives answer to  $\sqrt{8}$  as 4 rt 2;
- Simplify surd expressions involving squares (e.g.  $\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$ ).

**UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof**
**2a. Algebra: the basics, setting up, rearranging and solving equations** Teaching time  
9-11 hours

(N1, N3, N8, A1, A2, A3, A4, A5, A6, A7, A17, A20, A21)

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Use algebraic notation and symbols correctly;
- Know the difference between a term, expression, equation, formula and an identity;
- Write and manipulate an expression by collecting like terms;
- Substitute positive and negative numbers into expressions such as  $3x + 4$  and  $2x^3$  and then into expressions involving brackets and powers;
- Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g.  $l \times w$ ,  $v = u + at$ ;
- Simplify expressions by cancelling, e.g.  $\frac{4x}{2} = 2x$ ;
- Use instances of index laws for positive integer powers including when multiplying or dividing algebraic terms;
- Use instances of index laws, including use of zero, fractional and negative powers;
- Multiply a single term over a bracket and recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;
- Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to  $(2x + 3y)(3x - y)$ ;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise quadratic expressions of the form  $ax^2 + bx + c$ ;
- Factorise quadratic expressions using the difference of two squares;
- Set up simple equations from word problems and derive simple formulae;
- Understand the  $\neq$  symbol (not equal), e.g.  $6x + 4 \neq 3(x + 2)$ , and introduce identity  $\equiv$  sign;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects, including the kinematics formulae  $v = u + at$ ,  $v^2 - u^2 = 2as$ , and  $s = ut + \frac{1}{2} at^2$ ;
- Change the subject of a simple formula, i.e. linear one-step, such as  $x = 4y$ ;
- Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
- Simple proofs and use of  $\equiv$  in "show that" style questions; know the difference between an equation and an identity;
- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

**2b. Sequences**

(N8, N9, A23, A24, A25)

**Teaching time**

3-5 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences (including those involving numbers in standard form or index form);
- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Find and use (to generate terms) the  $n$ th term of an arithmetic sequence;
- Use the  $n$ th term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;
- Identify which terms cannot be in a sequence by finding the  $n$ th term;
- Continue a quadratic sequence and use the  $n$ th term to generate terms;
- Find the  $n$ th term of quadratic sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions ( $rn$  where  $n$  is an integer, and  $r$  is a rational number  $> 0$  or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
- Solve problems involving sequences from real life situations.

**UNIT 3: Averages and range, collecting data, representing data**

<b>3a. Averages and range</b> (G14, S2, S3, S4, S5)	<b>Teaching time</b> 3-5 hours
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**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Design and use two-way tables for discrete and grouped data;
- Use information provided to complete a two-way table;
- Sort, classify and tabulate data and discrete or continuous quantitative data;
- Calculate mean and range, find median and mode from a small data set;
- Use a spreadsheet to calculate mean and range, and find median and mode;
- Recognise the advantages and disadvantages between measures of average;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
  - find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Construct and interpret grouped frequency tables for continuous data:
  - for grouped data, find the interval which contains the median and the modal class;
  - estimate the mean with grouped data;
  - understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values.

<b>3b. Representing and interpreting data and scatter graphs</b> (S2, S3, S4, S5, S6)
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**Teaching time**  
4-6 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Know which charts to use for different types of data sets;
- Produce and interpret composite bar charts;
- Produce and interpret comparative and dual bar charts;
- Produce and interpret pie charts:
  - find the mode and the frequency represented by each sector;
  - compare data from pie charts that represent different-sized samples;
- Produce and interpret frequency polygons for grouped data:
  - from frequency polygons, read off frequency values, compare distributions, calculate total population, mean, estimate greatest and least possible values (and range);
- Produce frequency diagrams for grouped discrete data:
  - read off frequency values, calculate total population, find greatest and least values;
- Produce histograms with equal class intervals:
  - estimate the median from a histogram with equal class width or any other information, such as the number of people in a given interval;
- Produce line graphs:
  - read off frequency values, calculate total population, find greatest and least values;
- Construct and interpret time-series graphs, comment on trends;
- Compare the mean and range of two distributions, or median or mode as appropriate;

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- Recognise simple patterns, characteristics relationships in bar charts, line graphs and frequency polygons;
- Draw and interpret scatter graphs in terms of the relationship between two variables;
- Draw lines of best fit by eye, understanding what these represent;
- Identify outliers and ignore them on scatter graphs;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
- Understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' but merely 'no linear correlation';
- Explain an isolated point on a scatter graph;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

## UNIT 4: Fractions, percentages, ratio and proportion

### 4a. Fractions and percentages

(N1, N2, N3, N8, N10, N12, N13, R3, R9)

Teaching time

11-13 hours

#### OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express a given number as a fraction of another;
- Find equivalent fractions and compare the size of fractions;
- Write a fraction in its simplest form, including using it to simplify a calculation,  
e.g.  $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$ ;
- Find a fraction of a quantity or measurement, including within a context;
- Convert a fraction to a decimal to make a calculation easier;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions, including mixed numbers;
- Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
- Understand and use unit fractions as multiplicative inverses;
- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal and vice versa;
- Find the reciprocal of an integer, decimal or fraction;
- Convert between fractions, decimals and percentages;
- Express a given number as a percentage of another number;
- Express one quantity as a percentage of another where the percentage is greater than 100%;
- Find a percentage of a quantity;
- Find the new amount after a percentage increase or decrease;
- Work out a percentage increase or decrease, including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss;
- Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
- Find a percentage of a quantity using a multiplier and use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT;
- Use calculators for reverse percentage calculations by doing an appropriate division;
- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means  $2\frac{1}{2}$  times as big;
- Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations.

## 4b. Ratio and proportion

(N11, N12, N13, R2, R3, R4, R5, R6, R7, R8, R10)

Teaching time

5-7 hours

### OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express the division of a quantity into a number parts as a ratio;
- Write ratios in form  $1 : m$  or  $m : 1$  and to describe a situation;
- Write ratios in their simplest form, including three-part ratios;
- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known;
- Write a ratio as a fraction and as a linear function;
- Identify direct proportion from a table of values, by comparing ratios of values;
- Use a ratio to compare a scale model to real-life object;
- Use a ratio to convert between measures and currencies, e.g. £1.00 = €1.36;
- Scale up recipes;
- Convert between currencies.

**UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry**
**5a. Polygons, angles and parallel lines**  
(G1, G3, G4, G6, G11)

**Teaching time**  
5-7 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
- Understand 'regular' and 'irregular' as applied to polygons;
- Understand the proof that the angle sum of a triangle is  $180^\circ$ , and derive and use the sum of angles in a triangle;
- Use symmetry property of an isosceles triangle to show that base angles are equal;
- Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is  $360^\circ$ ; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is  $360^\circ$ ;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Use the angle sums of irregular polygons;
- Calculate and use the sums of the interior angles of polygons; use the sum of angles in a triangle and use the angle sum in any polygon to derive the properties of regular polygons;
- Use the sum of the exterior angles of any polygon is  $360^\circ$ ;
- Use the sum of the interior angles of an  $n$ -sided polygon;
- Use the sum of the interior angle and the exterior angle is  $180^\circ$ ;
- Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
- Calculate the angles of regular polygons and use these to solve problems;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Use angle facts to demonstrate how shapes would 'fit together', and work out interior angles of shapes in a pattern.

**5b. Pythagoras' Theorem and trigonometry**  
(N7, N8, N15, A4, A5, R12, G6, G20, G21)**Teaching time**  
5-7 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Understand, recall and use Pythagoras' Theorem in 2D;
- Given three sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
- Find the length of a shorter side in a right-angled triangle;
- Calculate the length of a line segment  $AB$  given pairs of points;
- Give an answer to the use of Pythagoras' Theorem in surd form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$ ; know the exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$ .

**UNIT 6: Real-life and algebraic linear graphs, quadratic and cubic graphs, the equation of a circle, plus rates of change and area under graphs made from straight lines**
**6a. Graphs: the basics and real-life graphs**  
 (N13, A8, A9, A10, A14, A15, R1, R11)

**Teaching time**  
 5-7 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance-time and velocity-time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.

**6b. Linear graphs and coordinate geometry**  
 (A9, A10, A12, A17, R10, R14)

**Teaching time**  
 7-9 hours

**OBJECTIVES**

By the end of the unit, students should be able to:

- Plot and draw graphs of  $y = a$ ,  $x = a$ ,  $y = x$  and  $y = -x$ , drawing and recognising lines parallel to axes, plus  $y = x$  and  $y = -x$ ;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form  $y = mx + c$  correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and  $y$ -intercept of a linear graph given by equations of the form  $y = mx + c$ ;
- Find the equation of a straight line from a graph in the form  $y = mx + c$ ;
- Plot and draw graphs of straight lines of the form  $y = mx + c$  with and without a table of values;
- Sketch a graph of a linear function, using the gradient and  $y$ -intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation  $ax + by = c$ ;
- Find the equation of a straight line from a graph in the form  $ax + by = c$ ;
- Plot and draw graphs of straight lines in the form  $ax + by = c$ ;
- Interpret and analyse information presented in a range of linear graphs:
  - use gradients to interpret how one variable changes in relation to another;
  - find approximate solutions to a linear equation from a graph;
  - identify direct proportion from a graph;
  - find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;

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- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when  $y = mx + c$  is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of  $m$  and a line perpendicular to this line will have a gradient of  $-\frac{1}{m}$ .

**6c. Quadratic, cubic and other graphs**  
(A11, A12, A14, A16, A18)**Teaching time**  
5-7 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Interpret graphs of quadratic functions from real-life problems;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$  using tables of values;
- Draw circles, centre the origin, equation  $x^2 + y^2 = r^2$ .

<b>UNIT 7: Perimeter, area and volume, plane shapes and prisms, circles, cylinders, spheres, cones; Accuracy and bounds</b>		
<b>7a. Perimeter, area and circles</b> (N8, N14, N15, A5, R1, G1, G9, G14, G16, G17, G18)	<b>Teaching time</b>	4-6 hours

**OBJECTIVES**

By the end of the unit, students should be able to:

- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
- Find the perimeter of a rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the perimeter of compound shapes made from triangles and rectangles;
- Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers;
- Recall the definition of a circle and name and draw parts of a circle;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference =  $2\pi r = \pi d$  and area of a circle =  $\pi r^2$ ) using a variety of metric measures;
- Use  $\pi \approx 3.142$  or use the  $\pi$  button on a calculator;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Calculate arc lengths, angles and areas of sectors of circles;
- Find radius or diameter, given area or circumference of circles in a variety of metric measures;
- Give answers to an appropriate degree of accuracy or in terms of  $\pi$ ;
- Form equations involving more complex shapes and solve these equations.

<b>7b. 3D forms and volume, cylinders, cones and spheres</b> (N8, N14, N15, A5, A21, G1, G12, G13, G14, G16, G17)	<b>Teaching time</b>
	6-8 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
- Draw sketches of 3D solids and identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm<sup>3</sup>;
- Use volume to solve problems;
- Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers;
- Use  $\pi \approx 3.142$  or use the  $\pi$  button on a calculator;
- Find the volume and surface area of a cylinder;
- Recall and use the formula for volume of pyramid;
- Find the surface area of a pyramid;

- Use the formulae for volume and surface area of spheres and cones;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
- Give answers to an appropriate degree of accuracy or in terms of  $\pi$ ;
- Form equations involving more complex shapes and solve these equations.

### 7c. Accuracy and bounds

(N15, N16)

**Teaching time**  
4-6 hours

#### OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the upper and lower bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of an expression involving the four operations;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
- Calculate the upper and lower bounds of calculations, particularly when working with measurements;
- Use inequality notation to specify an error interval due to truncation or rounding.

**UNIT 8: Transformations; Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings**
**8a. Transformations**

(R6, G5, G7, G8, G24, G25)

**Teaching time**

5-7 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations – know that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation,  $x = a$ ,  $y = a$ ,  $y = x$ ,  $y = -x$  and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also  $y = x$  and  $y = -x$ ;
- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;
- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

**8b. Constructions, loci and bearings**  
(R2, G2, G3, G12, G13, G15)

**Teaching time**  
6-8 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
  - bisect a given angle;
  - construct a perpendicular to a given line from/at a given point;
  - construct angles of  $90^\circ$ ,  $45^\circ$ ;
  - perpendicular bisector of a line segment;
- Construct:
  - a region bounded by a circle and an intersecting line;
  - a given distance from a point and a given distance from a line;
  - equal distances from two points or two line segments;
  - regions which may be defined by 'nearer to' or 'greater than'.
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

**UNIT 9: Algebra: Solving quadratic equations and inequalities, solving simultaneous equations algebraically**
**9a. Solving quadratics and simultaneous equations**
**Teaching time**

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Factorise quadratic expressions in the form  $ax^2 + bx + c$ ;
- Set up and solve quadratic equations;
- Solve quadratic equations by factorisation and completing the square;
- Solve quadratic equations that need rearranging;
- Solve quadratic equations by using the quadratic formula;
- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
  - linear / linear, including where both need multiplying;
  - linear / quadratic;
  - linear /  $x^2 + y^2 = r^2$ ;
- Set up and solve a pair of linear simultaneous equations in two variables, including to represent a situation

**9b. Inequalities****OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Solve two linear inequalities in  $x$ , find the solution sets and compare them to see which value of  $x$  satisfies both solve linear inequalities in two variables algebraically;
- Use the correct notation to show inclusive and exclusive inequalities.

**UNIT 10: Probability****Teaching Time**  
7-9 hours**OBJECTIVES**

By the end of the unit, students should be able to:

- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- List all outcomes for single events, and combined events, systematically;
- Draw sample space diagrams and use them for adding simple probabilities;
- Know that the sum of the probabilities of all outcomes is 1;
- Use  $1 - p$  as the probability of an event not occurring where  $p$  is the probability of the event occurring;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Find a missing probability from a list or two-way table, including algebraic terms;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events;
- Use a two-way table to calculate conditional probability;
- Use a tree diagram to calculate conditional probability;
- Use a Venn diagram to calculate conditional probability;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

**UNIT 11: Multiplicative reasoning: direct and inverse proportion, relating to graph form for direct, compound measures, repeated proportional change****Teaching Time**  
6-8 hours**OBJECTIVES**

By the end of the unit, students should be able to:

- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when  $A:B$  are in the ratio  $3:5$ ,  $A$  is  $\frac{3}{5}B$ . When  $4a = 7b$ , then  $a = \frac{7b}{4}$  or  $a:b$  is  $7:4$ ;
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;
- Work out the multiplier for repeated proportional change as a single decimal number;
- Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
- Understand and use compound measures and:
  - convert between metric speed measures;
  - convert between density measures;
  - convert between pressure measures;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of  $k$  in  $y = kx$ ;
- Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19);
- Relate algebraic solutions to graphical representation of the equations;
- Recognise when values are in inverse proportion by reference to the graph form;
- Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.

**UNIT 12: Similarity and congruence in 2D and 3D****Teaching Time**  
**5-7 hours****OBJECTIVES**

By the end of the unit, students should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Find missing lengths, areas and volumes in similar 3D solids;
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;
- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

<b>UNIT 13: Sine and cosine rules, <math>\frac{1}{2}ab \sin C</math>, trigonometry and Pythagoras' Theorem in 3D, trigonometric graphs, and accuracy and bounds</b>	
<b>13a. Graphs of trigonometric functions</b> (A8, A12, A13, G21)	<b>Teaching time</b> 5–7 hours

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees)  $y = \sin x$ ,  $y = \cos x$  and  $y = \tan x$  for angles of any size.
- Know the exact values of  $\sin \theta$  and  $\cos \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ, 60^\circ$  and  $90^\circ$  and exact value of  $\tan \theta$  for  $\theta = 0^\circ, 30^\circ, 45^\circ$  and  $60^\circ$  and find them from graphs.
- Apply to the graph of  $y = f(x)$  the transformations  $y = -f(x)$ ,  $y = f(-x)$  for sine, cosine and tan functions  $f(x)$ .
- Apply to the graph of  $y = f(x)$  the transformations  $y = f(x) + a$ ,  $y = f(x + a)$  for sine, cosine and tan functions  $f(x)$ .

<b>13b. Further trigonometry</b> (N16, A5, A8, G11, G20, G22, G23 )	<b>Teaching time</b> 8-10 hours
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**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Know and apply  $\text{Area} = \frac{1}{2}ab \sin C$  to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.

**UNIT 14: Statistics and sampling, cumulative frequency and histograms****14a. Collecting data**  
(S1)**Teaching time**  
5–7 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Specify the problem and plan:
  - decide what data to collect and what analysis is needed;
  - understand primary and secondary data sources;
  - consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it;
- Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative (see note);

**14b. Cumulative frequency, box plots and histograms**  
(S3, S4, S5)**Teaching time**  
5-7 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Use statistics found in all graphs/charts in this unit to describe a population;
- Know the appropriate uses of cumulative frequency diagrams;
- Construct and interpret cumulative frequency tables, cumulative frequency graphs/diagrams and from the graph:
  - estimate frequency greater/less than a given value;
  - find the median and quartile values and interquartile range;
- Compare the mean and range of two distributions, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
- Produce box plots from raw data and when given quartiles, median and identify any outliers;
- Know the appropriate uses of histograms;
- Construct and interpret histograms from class intervals with unequal width;
- Use and understand frequency density;
- From histograms:
  - complete a grouped frequency table;
  - understand and define frequency density;
- Estimate the mean and median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval.

**UNIT 15: Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics****Teaching time**  
6-8 hours**OBJECTIVES**

By the end of the unit, students should be able to:

- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots,  $y$ -intercept and turning point by completing the square;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Expand the product of more than two linear expressions;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Solve simultaneous equations graphically:
  - find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
  - find graphically the intersection points of a given straight line with a circle;
  - solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;
- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
  - for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of  $x^2 - 3x - 10 < 0$  as  $\{x: -3 < x < 5\}$ ;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

## UNIT 16: Circle theorems and circle geometry

### 16a. Circle theorems (G9, G10)

Teaching time  
4-6 hours

#### OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
- Prove and use the facts that:
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
  - the angle in a semicircle is a right angle;
  - the perpendicular from the centre of a circle to a chord bisects the chord;
  - angles in the same segment are equal;
  - alternate segment theorem;
  - opposite angles of a cyclic quadrilateral sum to  $180^\circ$ ;
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
  - circle theorems;
  - isosceles triangles (radius properties) in circles;
  - the fact that the angle between a tangent and radius is  $90^\circ$ ;
  - the fact that tangents from an external point are equal in length.

### 16b. Circle geometry (A16)

Teaching time  
4-6 hours

#### OBJECTIVES

By the end of the sub-unit, students should be able to:

- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
- Find the equation of a tangent to a circle at a given point, by:
  - finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
  - finding the gradient of the tangent perpendicular to it;
  - using the given point;
- Recognise and construct the graph of a circle using  $x^2 + y^2 = r^2$  for radius  $r$  centred at the origin of coordinates.

**UNIT 17: Changing the subject of formulae (more complex),  
algebraic fractions, solving equations arising from algebraic  
fractions, rationalising surds, proof****Teaching time**  
6-8 hours**OBJECTIVES**

By the end of the unit, students should be able to:

- Rationalise the denominator involving surds;
- Simplify algebraic fractions;
- Multiply and divide algebraic fractions;
- Solve quadratic equations arising from algebraic fraction equations;
- Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;
- Change the subject of a formula such as  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$ , where all variables are in the denominators;
- Solve 'Show that' and proof questions using consecutive integers ( $n, n + 1$ ), squares  $a^2, b^2$ , even numbers  $2n$ , odd numbers  $2n + 1$ ;
- Use function notation;
- Find  $f(x) + g(x)$  and  $f(x) - g(x)$ ,  $2f(x)$ ,  $f(3x)$  etc algebraically;
- Find the inverse of a linear function;
- Know that  $f^{-1}(x)$  refers to the inverse function;
- For two functions  $f(x)$  and  $g(x)$ , find  $gf(x)$ .

**UNIT 18: Vectors and geometric proof****Teaching time  
8-10 hours****OBJECTIVES**

By the end of the unit, students should be able to:

- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
- Understand that  $2\mathbf{a}$  is parallel to  $\mathbf{a}$  and twice its length, and that  $\mathbf{a}$  is parallel to  $-\mathbf{a}$  in the opposite direction.
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
- Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
- Find the length of a vector using Pythagoras' Theorem.
- Calculate the resultant of two vectors.
- Solve geometric problems in 2D where vectors are divided in a given ratio.
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

**UNIT 19: Direct and indirect proportion: using statements of proportionality, reciprocal and exponential graphs, rates of change in graphs, functions, transformations of graphs**
**19a. Reciprocal and exponential graphs; Gradient and area under graphs**
**Teaching time**  
 6-8 hours

(R11, R14, R15, R16, A7, A12, A13, A14, A15)

**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the reciprocal function  $y = \frac{1}{x}$  with  $x \neq 0$
- State the value of  $x$  for which the equation is not defined;
- Recognise, sketch and interpret graphs of exponential functions  $y = k^x$  for positive values of  $k$  and integer values of  $x$ ;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of  $f(x) + a$ , or  $f(x - a)$ :
  - apply to the graph of  $y = f(x)$  the transformations  $y = -f(x)$ ,  $y = f(-x)$  for linear, quadratic, cubic functions;
  - apply to the graph of  $y = f(x)$  the transformations  $y = f(x) + a$ ,  $y = f(x + a)$  for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance-time and velocity-time graphs:
  - for a non-linear distance-time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity-time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.

**19b. Direct and inverse proportion**  
(A21, R7, R10, R13, R14)**Teaching time**  
6-8 hours**OBJECTIVES**

By the end of the sub-unit, students should be able to:

- Recognise and interpret graphs showing direct and inverse proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for  $x$  squared and  $x$  cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use  $y = kx$  to solve direct proportion problems, including questions where students find  $k$ , and then use  $k$  to find another value;
- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.